

## SOME ALGEBRAIC POLYNOMIALS AND TOPOLOGICAL INDICES OF MÖBIUS LADDER

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**ABSTRACT.** In this paper we aim to compute some Zagreb type polynomials of Möbius Ladder. Moreover we compute redefined Zagreb indices of Möbius Ladder.

*Key words and phrases:* Zagreb index; Randić index; Polynomial; Möbius Ladder.

### 1. Introduction

The study of topological indices, based on distance in a graph, was effectively employed in 1947 in chemistry by Wiener [1]. He introduced a distance-based topological index called the "Wiener index" to correlate properties of alkenes and the structures of their molecular graphs. These indices play a vital role in computational and theoretical aspects of chemistry in predicting material properties [2, 3-8]. Several algebraic polynomials have useful applications in chemistry, [9, 10].

A graph  $G$  is an ordered pair  $(V, E)$ , where  $V$  is the set of vertices and  $E$  is the set of edges. A path from a vertex  $v$  to a vertex  $w$  is a sequence of vertices and edges that starts from  $v$  and stops at  $w$ . The number of edges in a path is called the length of that path. A graph is said to be connected if there is a path between any two of its vertices. The distance  $d(u, v)$  between two vertices  $u, v$  of a connected graph  $G$  is the length of a shortest path between them. Graph theory is contributing a lion's share in many areas such as chemistry, physics, pharmacy, as well as in industry [11]. We will start with some preliminary facts.

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The first and the second Zagreb indices are defined as

$$M_1(G) = \sum_{u \in V(G)} (d_u + d_v),$$

$$M_2(G) = \sum_{uv \in E(G)} d_u \times d_v,$$

For details see [12]. Considering the Zagreb indices, Fath-Tabar ([13]) defined first and the second Zagreb polynomials as

$$M_1(G, x) = \sum_{uv \in E(G)} x^{d_u + d_v},$$

and

$$M_2(G, x) = \sum_{uv \in E(G)} x^{d_u \cdot d_v}.$$

The properties of  $M_1(G, x)$ ,  $M_2(G, x)$  polynomials for some chemical structures have been studied in the literature [14,15].

After that, in [16], the authors defined the third Zagreb index

$$M_3(G) = \sum_{uv \in E(G)} (d_u - d_v),$$

and the polynomial

$$M_3(G, x) = \sum_{uv \in E(G)} x^{d_u - d_v}.$$

In the year 2016, [17] following Zagreb type polynomials were defined

$$M_4(G, x) = \sum_{uv \in E(G)} x^{d_u(d_u + d_v)},$$

$$M_5(G, x) = \sum_{uv \in E(G)} x^{d_v(d_u + d_v)},$$

$$M_{a,b}(G, x) = \sum_{uv \in E(G)} x^{ad_u + bd_v},$$

$$M'_{a,b}(G, x) = \sum_{uv \in E(G)} x^{(d_u + a)(d_v + b)}.$$

Ranjini et al. [18] redefines the Zagreb index, i.e, the redefined first, second and third Zagreb indices of graph  $G$ . These indicators appear as

$$ReZG_1(G) = \sum_{uv \in E(G)} \frac{d_u + d_v}{d_u d_v},$$

$$ReZG_2(G) = \sum_{uv \in E(G)} \frac{d_u \cdot d_v}{d_u + d_v},$$

and

$$ReZG_3(G) = \sum_{uv \in E(G)} (d_u + d_v)(d_u \cdot d_v).$$

In this paper we compute other Zagreb polynomials and Redefined Zagreb indices of Möbius Ladder. The Möbius ladder  $M_n$  which is a cubic circulant graph with an even number of vertices, formed from an  $n$ -cycle by adding edges (called "rungs") connecting opposite pair of vertices in the cycle. It is so-named because (with the exception of  $M_6 = K_{3,3}$  has exactly  $\frac{n}{2}$  4-cycles which link together by their shared edges to form a topological Möbius strip. Möbius ladders can also be viewed as a prism with one twisted edge. Two different views of Möbius ladders have been shown in Figure 1. Möbius ladders have many applications in chemistry, chemical stereography, electronics and computer science. For our convenience, we view the Möbius ladder  $M_n$  which is a cubic circulant graph with an even number of vertices, formed from an  $n$ -cycle by adding edges (called "rungs") connecting opposite pair of vertices in the cycle.

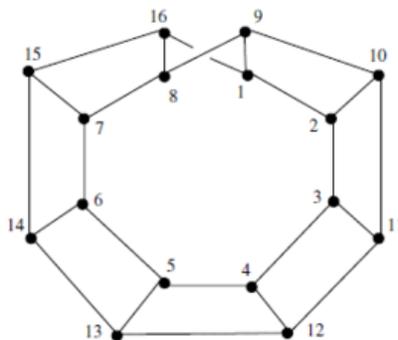


Figure 1. Möbius Ladder  $M_{16}$

## 2. Computational Results

Let In this section, we present our computational results.

**Theorem 2.1.** *Let  $M_n$  be the Möbius Ladder. Then*

- (1)  $M_3(M_n, x) = 3n$ ,
- (2)  $M_4(M_n, x) = 3nx^{18}$ ,
- (3)  $M_5(M_n, x) = 3nx^{18}$ ,
- (4)  $M_{a,b}(M_n, x) = 3nx^{3a+3b}$ ,
- (5)  $M'_{M_n,b}(G, x) = 3nx^{(3+a)(3+b)}$ .

*Proof.* Let  $M_n$  be the Möbius Ladder. It is clear that  $M_n$  has only one partition of vertex set i.e,

$$V_1 = \{v \in V(M_n) : d_v = 3\},$$

The edge set of  $M_n$  has following one partition,

$$E_1 = E_{3,3} = \{e = uv \in E(M_n) : d_u = 3, d_v = 3\},$$

Now,

$$(1) \quad |E_1(M_n)| = 3n,$$

$$\begin{aligned} M_3(G, x) &= \sum_{uv \in E(M_n)} x^{(d_u - d_v)} \\ &= \sum_{uv \in E_1(M_n)} x^{(3-3)} \\ &= |E_1(M_n)| \\ &= 3n. \end{aligned}$$

(2)

$$\begin{aligned} M_4(G, x) &= \sum_{uv \in E(M_n)} x^{d_u(d_u + d_v)} \\ &= \sum_{uv \in E_1(M_n)} x^{3(3+3)} \\ &= |E_1(M_n)| x^{18} \\ &= 3nx^{18}. \end{aligned}$$

(3)

$$\begin{aligned} M_5(G, x) &= \sum_{uv \in E(M_n)} x^{d_v(d_u + d_v)} \\ &= \sum_{uv \in E_1(M_n)} x^{3(3+3)} \\ &= |E_1(M_n)| x^{18} \\ &= 3nx^{18}. \end{aligned}$$

(4)

$$\begin{aligned} M_{a,b}(G, x) &= \sum_{uv \in E(M_n)} x^{(ad_u + bd_v)} \\ &= \sum_{uv \in E_1(M_n)} x^{(3a + 3b)} \\ &= |E_1(M_n)| x^{3a+3b} \\ &= 3nx^{3a+3b}. \end{aligned}$$

(5)

$$M'_{a,b}(G, x) = \sum_{uv \in E(M_n)} x^{(d_u + a)(d_v + b)}$$

$$\begin{aligned}
&= \sum_{uv \in E_1(M_n)} x^{(3+a)(3+b)} \\
&= |E_1(M_n)| x^{(3+a)(3+b)} \\
&= 3nx^{(3+a)(3+b)}.
\end{aligned}$$

□

**Theorem 2.2.** *Let  $M_n$  be the Möbius Ladder. Then,*

- (1)  $ReZG_1(M_n) = 2n$ ,
- (2)  $ReZG_2(M_n) = \frac{9}{2}n$ ,
- (3)  $ReZG_3(M_n) = 162n$ .

*Proof.* (1)

$$\begin{aligned}
ReZG_1(M_n) &= \sum_{uv \in E(M_n)} \frac{d_u + d_v}{d_u d_v} \\
&= \sum_{uv \in E_1(M_n)} \frac{d_u + d_v}{d_u d_v} \\
&= \sum_{uv \in E_1(M_n)} \frac{3 + 3}{3 \cdot 3} \\
&= |E_1(M_n)| \frac{6}{9} \\
&= (3n) \frac{6}{9} \\
&= 2np.
\end{aligned}$$

(2)

$$\begin{aligned}
ReZG_2(G) &= \sum_{uv \in E(M_n)} \frac{d_u \cdot d_v}{d_u + d_v} \\
&= \sum_{uv \in E_1(M_n)} \frac{d_u \cdot d_v}{d_u + d_v} \\
&= \sum_{uv \in E_1(M_n)} \frac{3 \cdot 3}{3 + 3} \\
&= (3n) \frac{3}{2} \\
&= \frac{9}{2}n.
\end{aligned}$$

(3)

$$ReZG_2(G) = \sum_{uv \in E(M_n)} (d_u \cdot d_v)(d_u + d_v)$$

$$\begin{aligned}
&= \sum_{uv \in E_1(M_n)} (d_u \cdot d_v)(d_u + d_v) \\
&= \sum_{uv \in E_1(M_n)} (3 \cdot 3)(3 + 3) \\
&= |E_1(G)| \cdot 54 \\
&= 3n(54) \\
&= 162n.
\end{aligned}$$

□

### Competing Interests

The author do not have any competing interests in the manuscript.

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